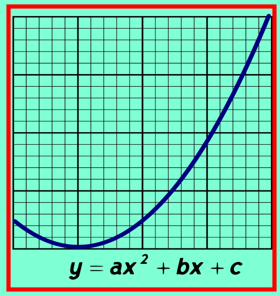
Math 125

Fall 2021

Lecture 38



Simplify

1) $\underbrace{(\sqrt{13}+\sqrt{7})(\sqrt{13}-\sqrt{7})}_{\text{Conjugates}} = (\sqrt{13})^2 - \cancel{\sqrt{91}} + \cancel{\sqrt{91}} - (\sqrt{7})^2$

$= 13 - 7 = \boxed{6}$

2) $(2\sqrt{3}-\sqrt{10})^2 = (2\sqrt{3}-\sqrt{10})(2\sqrt{3}-\sqrt{10})$

$= 4\sqrt{9} - 2\sqrt{30} - 2\sqrt{30} + \sqrt{100}$

$= 4\cdot 3 - 4\sqrt{30} + 10$

$= \boxed{22 - 4\sqrt{30}}$

3) $\dfrac{3+4i}{4-2i}$

$= \dfrac{3+4i}{4-2i}\cdot\dfrac{4+2i}{4+2i}$

$= \dfrac{12+6i+16i+8i^2}{16+\cancel{8i}-\cancel{8i}-4i^2} = \dfrac{12+22i-8}{16+4} = \dfrac{4+22i}{20}$

$= \dfrac{4}{20} + \dfrac{22}{20}i = \boxed{\dfrac{1}{5} + \dfrac{11}{10}i}$

Simplify

1) $i^{53} = i^{52} \cdot i = (i^2)^{26} \cdot i = (-1)^{26} \cdot i = 1 \cdot i = \boxed{i}$

2) $i^{35} = i^{34} \cdot i = (i^2)^{17} \cdot i = (-1)^{17} \cdot i = -1 \cdot i = \boxed{-i}$

3) $\sqrt{-72} \cdot \sqrt{-50} = \sqrt{36}\sqrt{2}\sqrt{-1} \cdot \sqrt{25} \cdot \sqrt{2} \cdot \sqrt{-1}$

$= 6\sqrt{2}\, i \cdot 5\sqrt{2}\, i$

$= 30\sqrt{4}\, i^2$

$= 30 \cdot 2 \cdot (-1) = \boxed{-60}$

Rationalize the deno.:

1) $\frac{15}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{15\sqrt{5}}{\sqrt{25}}$

$= \frac{15\sqrt{5}}{5} = \boxed{3\sqrt{5}}$

2) $\frac{4}{\sqrt{5}+1} \cdot \frac{\sqrt{5}-1}{\sqrt{5}-1}$

$= \frac{4(\sqrt{5}-1)}{\underbrace{\sqrt{25}}_{5} \cancel{-\sqrt{5}} \cancel{+\sqrt{5}} \underbrace{-1}_{-1}}$

$= \frac{\cancel{4}(\sqrt{5}-1)}{\cancel{4}} = \boxed{\sqrt{5}-1}$

Solve & check:

$\sqrt{x+2} - \sqrt{x+1} = 1$

$\sqrt{x+2} = 1 + \sqrt{x+1}$

$\left(\sqrt{x+2}\right)^2 = \left(1 + \sqrt{x+1}\right)^2$

$x+2 = (1+\sqrt{x+1})(1+\sqrt{x+1})$

$x+2 = 1 + \sqrt{x+1} + \sqrt{x+1} + (\sqrt{x+1})^2$

$x \cancel{+2} = \cancel{1} + 2\sqrt{x+1} + x \cancel{+1}$

$0 = 2\sqrt{x+1}$

Divide by 2

$\sqrt{x+1} = 0$

$\left(\sqrt{x+1}\right)^2 = 0^2$

$x+1 = 0$

$x = -1$

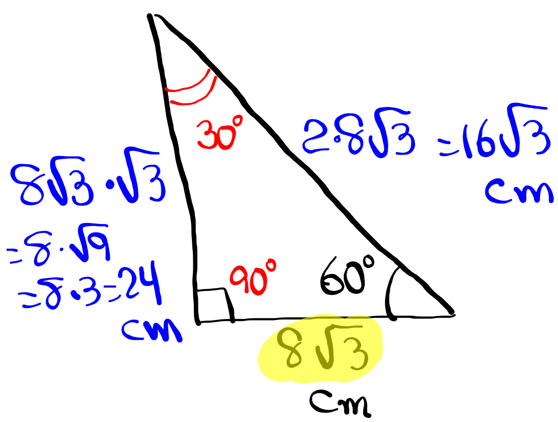
Check

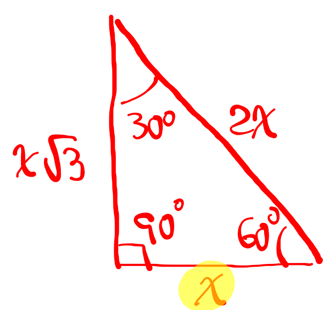
$\sqrt{-1+2} - \sqrt{-1+1} = 1$

$\sqrt{1} - \sqrt{0} = 1 \quad \boxed{1=1}$ ✓

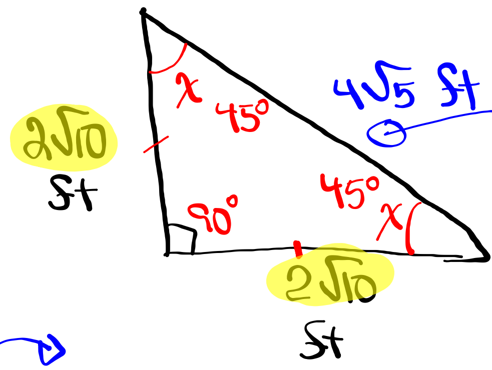
$\{-1\}$

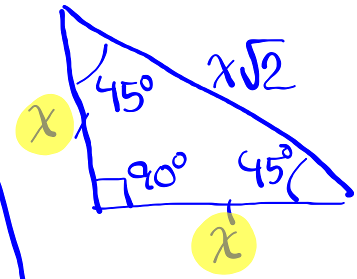
Find missing sides and missing angle:





Find missing side & missing angles:





$90^\circ + x + x = 180^\circ$

$2x = 90^\circ$

$x = 45^\circ$

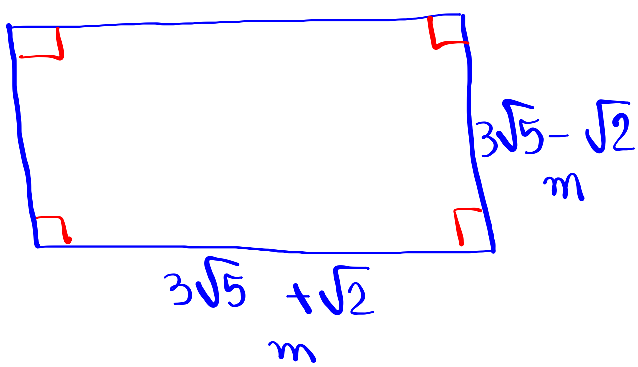
$2\sqrt{10}\sqrt{2}$

$= 2\sqrt{20} = 2\sqrt{4}\sqrt{5}$

$= 2 \cdot 2\sqrt{5}$

$= \boxed{4\sqrt{5}}$

Find area & Perimeter



$A = LW$

$= (3\sqrt{5} + \sqrt{2})(3\sqrt{5} - \sqrt{2})$

$= 9\sqrt{25} - 3\sqrt{10} + 3\sqrt{10} - \sqrt{4}$

$= 9 \cdot 5 - 2$

$= \boxed{43 \text{ m}^2}$

$P = 2L + 2W$

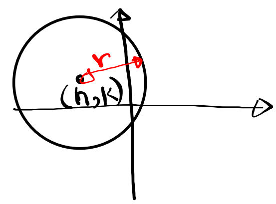
$= 2(3\sqrt{5} + \sqrt{2}) + 2(3\sqrt{5} - \sqrt{2})$

$= 6\sqrt{5} + 2\sqrt{2} + 6\sqrt{5} - 2\sqrt{2} = \boxed{12\sqrt{5} \text{ m}}$

Circle:

Center (h,k)

Radius r

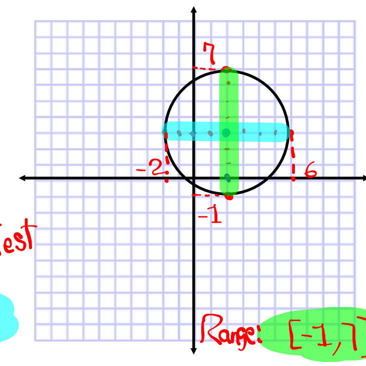


Equation $(x-h)^2+(y-k)^2=r^2$

Given $(x-2)^2+(y-3)^2=16$

$r^2=16$

$\boxed{r=4}$



Not a Function
→ It Fails the Vertical line Test

Domain: $[-2,6]$

Range: $[-1,7]$

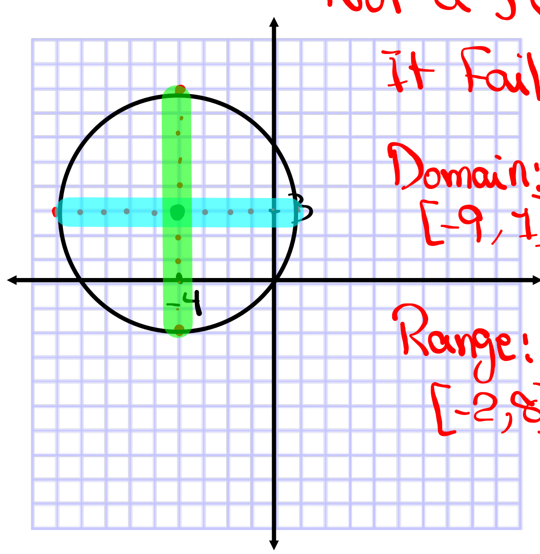
$(x+4)^2+(y-3)^2=25$ $\qquad r^2=25$

$(x-h)^2+(y-k)^2=r^2$

$h=-4$

$k=3$

$r=5$



Not a Function
It Fails V.L.T.

Domain: $[-9,1]$

Range: $[-2,8]$

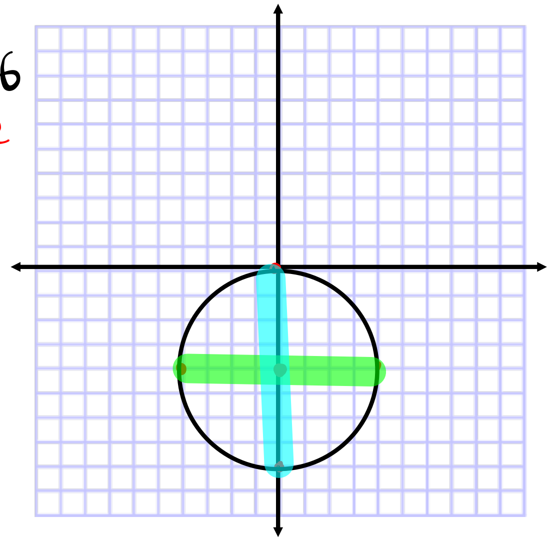
Given $x^2 + (y+4)^2 = 16$

$(x-h)^2 + (y-k)^2 = r^2$

h = 0

Draw

K = -4



r = 4

Domain [-4,4]

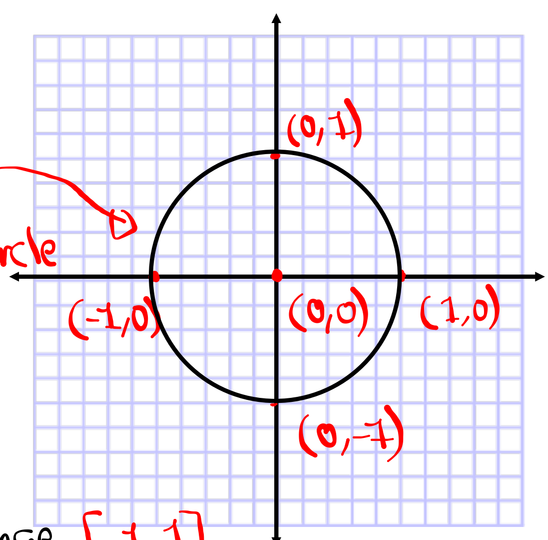
Range [-8,0]

Given $x^2 + y^2 = 1$

h = 0

K = 0

Unit Circle



r = 1

Draw

Domain [-1,1] Range [-1,1]

Write $x^2-8x+16+y^2+6y+9=36$

in $(x-h)^2+(y-k)^2=r^2$ form.

$(x-4)^2+(y+3)^2=6^2$

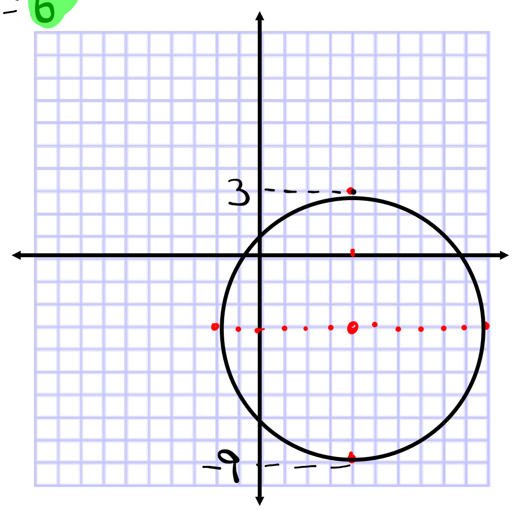
Center $(4,-3)$

Radius $r=6$

Draw

Domain $[-2,10]$

Range $[-9,3]$



Solve

$$-1\begin{cases} x^2+y^2=10 \\ 4x^2+y^2=13 \end{cases} \Rightarrow \begin{cases} -x^2-y^2=-10 \\ 4x^2+y^2=13 \end{cases}$$

$3x^2=3$

Divide by 3

$x^2=1$

$\boxed{x=\pm 1}$

Now

$x^2+y^2=10$

$1+y^2=10$

$y^2=9$

$\boxed{y=\pm 3}$

4 Answers

$(1,3),\ (1,-3),\ (-1,3),\ (-1,-3)$

Solve

$$\begin{cases} x^2 + y^2 = 26 \\ x^2 - y^2 = 24 \end{cases}$$

$$2x^2 = 50$$

$$x^2 = 25$$

$$\boxed{x = \pm 5}$$

$$25 + y^2 = 26$$

$$y^2 = 1$$

$$\boxed{y = \pm 1}$$

Here are the answers:

$(5, 1)$, $(-5, 1)$, $(5, -1)$, $(-5, -1)$

Draw $x^2 + y^2 = 25$ and $y = \frac{2}{3}x + 1$.

Shade inside the Circle but above the line.

Center $(0,0)$

Radius 5

Y-Int $(0, 1)$

$m = \frac{2}{3}$